16/11/2015 الـــتـــنـــيـــم م. جمال محاضرة [4]

Mean and Variance :-

⇒ Discrete random Variable

The complete description of a discrete random variable requires specifying the opposite values that the random variable may take and the probability associated with each value (probability and distribution)

x	1	2	3	4	5	6
P(x)	✓	✓	✓	✓	✓	✓

* Mean :- expectation, expected value.

$E(x)$, μ_x , μ

- first moment of the origin $= \sum_x x p(x)$
- second " " " " $= \sum_x x^2 p(x)$
- rth " " " " $= \sum_x x^r p(x)$

For cont. ⇒ replace sum (Σ) with integration ($\int$)

* Variance

1st moment about the mean

$= E(x-\mu) = \sum_x (x-\mu)\, p(x)$

2nd moment about the mean

$= E(x-\mu)^2 = \sum_x (x-\mu)^2 p(x)$

Var σ^2

rth moment around the mean

$= E(x-\mu)^r = \sum_x (x-\mu)^r p(x)$

Standard Deviation ~ S.D.

$\sigma = \sqrt{Var(x)}$

Ex: prove that $Var(x) = E(x^2) - \mu^2$

$$Var(x) = \sum_x (x-\mu)^2 \, p(x)$$

$$= \sum_x (x^2 - 2\mu x + \mu^2) \, p(x)$$

$$= \sum_x x^2 p(x) - 2\mu \sum_x x p(x) + \mu^2 \sum_x p(x)$$

$$= \sum_x x^2 p(x) - 2\mu^2 + \mu^2$$

$$= E(x^2) - \mu^2$$ ⇐ (بالتعويض)

⇒ For Continuous Random Variable :-

* Mean

1st moment about the origin $= E(x) = \int_{-\infty}^{\infty} x p(x) dx$

2nd " " " " $= E(x^2) = \int_{-\infty}^{\infty} x^2 p(x) dx$

* Variance

1st moment about mean $= E(x-\mu) = \int_{-\infty}^{\infty} (x-\mu) p(x) dx$

2nd " " " $= E(x-\mu^2) = \int_{-\infty}^{\infty} (x-\mu)^2 p(x) dx$

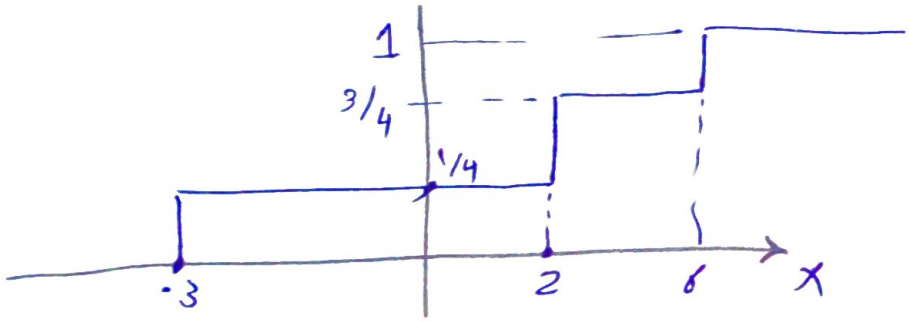
Sheet 4

(7) plot the graph of F(x) for the variable X with density

x	-3	2	6
p(x)	1/4	1/2	1/4

$F(-\infty) = 0$, $F(-3) = p(-3) = 1/4$

$F(2) = p(2) + p(-3) = 3/4$, $F(6) = 1$



Sheet 5

(1) X equal the no. of successful cures out of the five.

X	0	1	2	3	4	5
P(x)	0.002	0.029	0.132	0.309	0.36	0.168

(a) $\mu \Rightarrow \mu = \sum_x x\,p(x) = 0(0.002) + 1(0.029) + 2(0.132) + 3(0.309) + 4(0.36) + 5(0.168) = 3.5$

(b) σ

$$\Rightarrow \sigma^2 = \sum (x-\mu)^2 p(x) = (0-3.5)^2(0.002) + (1-3.5)^2\,0.029 + (2-3.5)^2 + (3-3.5)^2\,0.309 + (4-3.5)^2\,0.36 + (5-3.5)^2\,0.168 = \checkmark$$

$$\sigma = \sqrt{\sigma^2} = 1.02$$

[2] Find the expectation, variance, S.D.

iv) $p(x) = \begin{cases} \frac{2}{25}x & 0 \le x \le 5 \\ 0 & \text{elsewhere} \end{cases}$

* $\mu = \int_{-\infty}^{\infty} x\, p(x)\, dx$

$= \int_0^5 \frac{2}{25} x^2 dx = \frac{2}{25} x^3/3 \Big|_0^5 = \frac{10}{3}$

* $Var(x) = E(x) - \mu^2$

$E(X^2) = \int_{-\infty}^{\infty} x^2 p(x)\, dx = 12.5$

$Var = 12.5 - \left(\frac{10}{3}\right)^2 = \checkmark$

* $\sigma = \sqrt{var} = \checkmark$